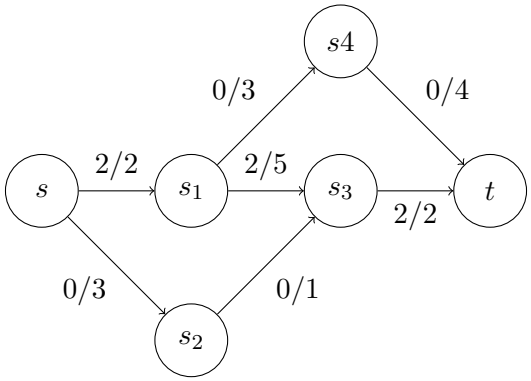
Technische Universität München
Institut für Informatik
Theoretical Computer Science

Fundamental Algorithms 13 - Solution Examples

Exercise 1 (Ford-Fulkerson)

Consider the following flow network G during the execution of Ford-Fulkerson. The edge labels, written f/c, denote the flow f and capacity c of the respective edge. Draw the residual graph G_f, find an augmenting path, and apply the Ford-Fulkerson operations. Continue the algorithm until you identified the maximum flow in G.



Solution:

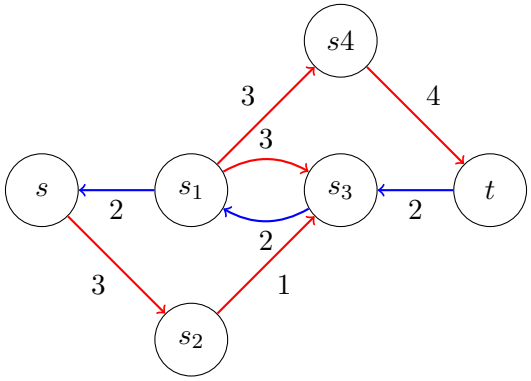


Figure 1: The residual graph.

The algorithm actually is finished now, with the maximum flow equal 3.

Exercise 2 (Marriage)

The *maximum bipartite matching* problem is defined as follows. Given a bipartite graph $((U, V), E)$, i.e. a graph where edges are exclusively between U and V, find a largest set of edges such that no two edges share a vertex. Informally, this can be interpreted as *marriage problem*: The vertex sets represent the males and

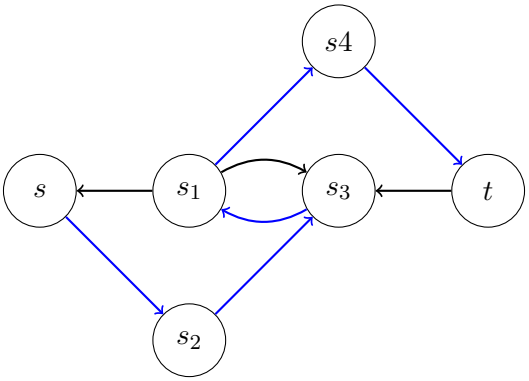


Figure 2: The augmenting path.

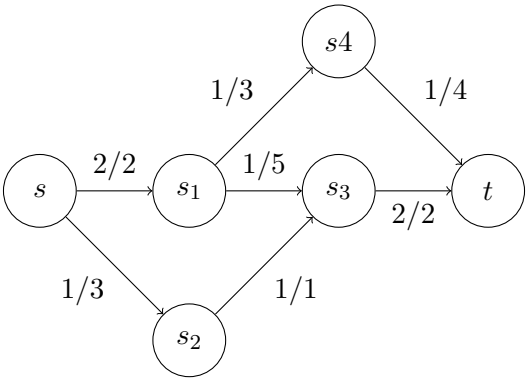


Figure 3: The augmented graph.

females of a particular population, edges represent mutual interest. Now, we want to identify the maximum amount of marriages.

Think about how to apply Ford-Fulkerson to solve this problem.

Solution:
Introduce a *super-source* s, connected to all nodes in U, and a *super-sink* t, connected to all nodes in V. Now, applying Ford-Fulkerson to this graph immediately yields the solution.